Structural Response to Detonating High-Explosive Projectiles

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A description is given of the BR-1 (Blast Response) finite element computer code that was developed to predict the local damage and transient response of metallic aircraft structural compartments subjected to blast pressures and fragments from the detonation in air of high-explosive (HE) projectiles within the compartment. The structural analysis is limited to isotropic materials and unstiffened or skin-rib-stringer type construction. Large deflections, large strains, nonlinear stress-strain relations, and ultimate strain material failures are accounted for in the structural response theory that was assembled in the development of the BR-1 code. The impulse imparted to the structural compartment and the loss of structural mass and stiffness because of penetration of fragments of certain HE projectiles are accounted for. An available blast pressure loading computer program was used as the basis of the blast pressure loading subroutine imbedded in the BR-1 code. Sample problems of structural response to blast loading that were executed with the BR-1 code are discussed and the prominent role of the membrane stresses in the response is noted.

Nomenclature

A_f	= average presented area of fragment
$\{C\}$	= equivalent force matrix
C_i	= empirical constants of the target ($i = 1, 2, 3, 4$)
$\{F\}$	= external and body force matrix
$[H]$	= a stiffness matrix for nonlinear analysis
$[M]$	= mass matrix
m	= fragment weight
$\bar{m}$	= average fragment weight
N_0	= total number of fragments
$N(m)$	= number of fragments with weight greater than or equal to m
$\{P\}$	= nodal force matrix
ΔP	= instantaneous incident overpressure
ΔP_i	= peak incident shock overpressure
$\{q^*\}$	= generalized displacement matrix
R	= distance from explosion (the minimum R for calculations is 65 cm)
s	= stress
t	= time measured from shock arrival
t_d	= positive phase duration
t_p	= target plate thickness
V	= minimum fragment velocity for perforation
x,y	= Cartesian coordinates
θ	= obliquity angle of impacting fragment

Introduction

THIS paper is addressed to the development and application of the BR-1 computer program for predicting the transient structural response (i.e., deflections, strains, and stresses up to and including ultimate strength failures) and local damage to metallic structural compartments subjected to blast pressures and fragments from high-explosive (HE) projectiles that penetrate into, and detonate in air within, the compartment.[1,2] The BR-1 code is expected to be used principally in aircraft survivability/vulnerability studies in which local structural failures and damage resulting from the blast are of interest because of their influence on the structural load redistribution (because of blast damage) that may result in a catastrophic vehicle failure under some flight conditions.

Structural design approaches to account for internal blast effects have generally been of a type which utilized the correlation of empirical data in the form outlined in Ref. 3. Such methods are still utilized, but they are updated with more recent information regarding blast effects when appropriate (e.g., Ref. 4) for the evaluation of airframe vulnerability of combat aircraft.

Numerical methods based on either the finite-difference method or the finite-element have been used to obtain the dynamic response of structures to impulsive loading. Witmer and his associates developed a prediction technique based on the finite difference method to investigate large-deformation, elastic-plastic, transient and permanent deflection responses to pressure loading.[5,6] Subsequently, Wu and Witmer developed a finite-element method and analyzed the large-deflection transient responses of a beam and a ring and included elastic-plastic, strain-hardening, and strain-rate material behavior.[7]

Experiments have produced useful information on structural response to impulsive loading. Jones et al. undertook an experimental program to investigate the dynamic behavior of wide beams and rectangular flat plates that were fully clamped and axially restrained at two ends.[8] They concluded that rigid plastic theories for fully clamped beams give reasonable engineering estimates of the permanent transverse deflections provided that the influence of geometry changes and material strain-rate sensitivity were retained when appropriate. Beynet and Plunkett conducted an analytic and experimental investigation on the deformation of thin plates struck by blunt projectiles that did not perforate the plate but produced large transverse deflections.[9] They showed that for the thin plates under transverse impact producing plastic deformations, the plastic deformations were governed by membrane stress, and a small-strain large-deflection theory was adequate for predicting the deflections of 2024-T3, 6061-T6, 7075-T6 aluminum alloy plates.

In a series of publications,[10-13] many items were investigated pertaining to fragmentation. Analytic models were developed that define the dynamic behavior of armor-piercing projectiles, their fragments, and secondary particles generated during oblique penetration of multiple-plate arrays.

Proctor developed the BLAST computer code to describe the shock and blast loading characteristics of the detonation

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of a HE projectile internal to an aircraft structure.[14,15] Portions of the BLAST code are used as a subroutine within the BR-1 computer code for predicting blast pressures.

Theoretical Features Incorporated into the BR-1 Code

The BR-1 finite element code is based on the assumed displacement method and was developed to determine effects of internal blast on aircraft structures.[1] Very soon after an internal blast occurs, the structure is loaded dynamically by pressure from the blast and fragments (from the projectile) that impact and may in many instances penetrate the structure. In many instances, the dynamic loading produces a transient structural response that may be characterized by large deflections, large strains, high strain rates, and structural failure.

The BR-1 code is operational on the CDC 6600 and IBM 370/165 systems. The principal features of the structural response theory, the fragmentation theory, and blast pressure loading theory are discussed below.

Structural Response Theory

Nonlinear differential equations govern the structural transient responses. Due to the nonlinearities of the system it becomes imperative to pursue the response solutions by numerical methods. The BR-1 code is based on "two dimensional" theory (i.e., the structure and structural deformations are defined in terms of two independent spatial variables) of structures in three-dimensional space. The BR-1 code accounts for large deflections, finite strains, and nonlinear material properties. The finite elements are restricted to flat rectangular and triangular plate elements and straight finite beam elements.[1,16] The constitutive equations were developed for isotropic materials and incorporate the von Mises yield criterion, the Prandtl-Reuss normal flow rule, and isotropic work hardening.

The strain-displacement equations that were used in the development of the BR-1 code were chosen to avoid the limitations that deflections be small compared to the plate surface dimensions. The equations are deemed suitable for strains up to approximately 10% and rotations up to approximately 20%. Insofar as finite strains are concerned, the BR-1 code was developed under the assumption, which is deemed acceptable when strains are limited to the order of 10%, that the product of strains is negligible compared to unity. With this finite strain approach, the Kirchhoff stress required in the strain energy term was properly obtained and the von Mises yield criterion was written in true invariant form with the stress definition based on the deformed cross-sectional areas. This same finite strain approach (without the BR-1 simplification that the finite strains be moderate, i.e. less than 10%) has been successfully backed up by experimental verification with straining into the failure region of strains under the condition of uniaxial tension.[17]

The scheme to determine the transient structural response in the BR-1 code is similar to the method recently presented by Wu and Witmer.[7] Using the Principle of Virtual Work together with D'Alembert's Principle, the dynamic equations of equilibrium are obtained. In particular, the dynamic equations of equilibrium which give rise to the transient response are of the form

$$[M]\{\ddot{q}^*\} = \{C\} \tag{1}$$

where

$$\{C\} = \{F\} - \{P\} - [H]\{q^*\} \tag{2}$$

The method of obtaining the $\{F\}$, $\{P\}$, $[H]$, and $[M]$ matrices is given in detail in Ref. 1; the lumped mass approach is used in developing $[M]$.

Leech had presented a criterion that would be applicable in determining the maximum time increment for the time integration in the BR-1 code.[18] An iterative technique was developed for and incorporated into the BR-1 code to automate the utilization of this criterion so that a numerically stable time increment for the numerical integration will be computed by the BR-1 code at the user's request.

The integration of Eq. (1) is carried out numerically in the BR-1 code with the use of the central-finite-difference method to obtain displacement increments at the end of each time increment. From the incremental displacements and the strain-displacement equations, incremental strains are computed. The incremental strains are used in combination with the constitutive relations to determine incremental stresses. The total displacements, strains, and stresses are then updated by the BR-1 code at the end of each time increment.

Fragmentation

In the BR-1 code, fragmentation is considered in the following manner. The fragmentation characteristics of three HE projectiles are imbedded into the BR-1 code.[1,2] In particular, the static velocity, the mean static direction, the number of fragments, and the mass distribution for the fuze, the fuze attachment, the side portion, and the base of the HE projectiles are imbedded in the BR-1 code.

The velocity of the fragments is vectorally added in the BR-1 code to the velocity of the projectile relative to the target at the time of detonation to determine the relative velocity of the fragments to the target structure. For fragments emanating from the side of the projectile, the code automatically determines which finite elements in the structure are struck. For fragments emanating from the nose or base of the projectile, the user has to specify through the input data the elements struck. The BR-1 code determines what percentage of the fragments penetrate the structure, the impulse imparted to the finite elements because of the momentum change of the fragments as they encounter and/or penetrate the structure, and the loss in strain energy carrying capability and mass of the structure because of holes created by fragments that penetrate the structure.

The criterion for plate perforation by a fragment is given by the empirical equation[19]

$$V_c = 10^{C_1} t_p A_f^{C_2} m^{C_3} (\sec\theta)^{C_4} \tag{3}$$

Since most of the mass of the totality of the fragments sprays from the side of the projectiles under consideration, the determination of side spray characteristics is of utmost concern. For the side spray, the relation between the number of fragments and the size of the fragments is assumed to be given by the Mott size distribution equation,[20] namely,

$$N(m) = N_0 \exp - (2m/\bar{m})^{1/2} \tag{4}$$

Blast Pressure Loading

The BLAST code was developed to describe the shock and blast loading characteristics of the detonation of a HE projectile internal to an aircraft structure.[15] The BLAST code is used as the basis of a subroutine in the BR-1 code to determine the transient pressure (or impulse) and quasistatic overpressure resulting from the internal detonation.

The BLAST subroutine of the BR-1 code analytically divides the internal explosion into two damaging mechanisms: the shock wave, and the confined-explosion gas pressure (i.e. the quasistatic overpressure). For the shock wave the code generates the incident and normally reflected pressure-time history and impulse for the positive phase duration at specified distances (namely, the distances between the center of the explosion and the nodes of the modeled structure). Existing state-of-the-art explosion theory and experimental data were used to develop the shock calculation model.

The code reduces the shock calculation for all cases to the reference data from a freefield, bare spherical 1-lb TNT explosion at sea-level ambient conditions.[14] The pressure-time history during the positive phase duration is given by the empirical equation

$$\Delta P/\Delta P_i = (1-t/t_d)\exp - t/t_d\left[1+\frac{(228/R-0.95)}{(0.5\ t/t_d)}\right] \tag{5}$$

The code has provisions for converting the incident overpressure to normally reflected pressure; the normally reflected pressures are the pressures applied to panel element surfaces.

The quasistatic overpressure that is computed in the BLAST and the BR-1 codes account for the chemical reaction of the explosive with the surrounding air until the final gas temperature is reached. The code uses the final temperature and amount of gas in the structural volume to determine the final pressure, i.e. the quasistatic overpressure. The BLAST code has the capability of calculating the variation of the overpressure with time for venting due to holes in the entry and exit walls, but this feature of the BLAST code is not incorporated into the BR-1 code, because venting effects are not expected to be important in the transient response of the order of one millisecond, which is characteristic of problems that the BR-1 code is expected to solve.

Termination of Computer Run

The BR-1 code continues a response determination until the computer run is terminated because either 1) the total number of specified maximum time increments are executed, 2) the restrictions on computer computation time are violated, or 3) the failure of the structure occurs. The user has three options of defining failure of the structure that will result in a computer run termination. These options are point failure, station failure, and node failure, and their definitions are given below.

The stresses and strains for each finite element of the structure are defined only at discrete points of the cross section corresponding to discrete midsurface (midline) Gaussian stations.[2] Point failure is defined as the ultimate failure of the material at a point, and point failure is to be interpreted then as corresponding to failure at one of the discrete points of the element where the stresses and strains are being monitored. Station failure is defined as failure at all monitored points (through the thickness) at a Gaussian station. Node failure is defined as occurring when station failures exist in all finite elements tieing into a node.

After several point and/or station failures occur, the capability of the assumed displacement functions to represent the actual deflection shape becomes questionable. Therefore, the user of the BR-1 code should exercise particular caution in using results obtained when there are several station failures.

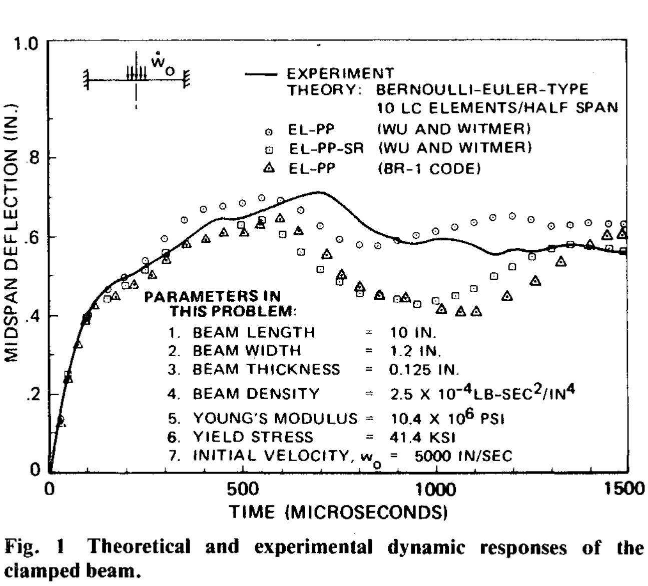

Fig. 1 Theoretical and experimental dynamic responses of the clamped beam.

Sample Problems

Some sample problems have been formulated and executed with the BR-1 computer code. Discussions of three of the sample problems that were executed on the IBM 370/165 system are presented below.

Beam Response to Impulsive Pressure over a Portion of the Beam

A sample case of a beam loaded impulsively over 20% of its length at the center of the beam was executed with the BR-1 code and compared with results with and without strain rate effects given by Wu and Witmer.[21] The experimental results, the Wu and Witmer theoretical results, and the BR-1 code results are included in Fig. 1. (In Fig. 1, EL-PP means an elastic-perfectly plastic material with strain rate effects.) All three of the theoretical curves in the figure agree fairly well with the experimental results. Exact agreement between theoretical results from the BR-1 code and the results from Ref. 21 was not expected because of some differences in the finite-element structural models.

Enclosed Compartment Responses to Internal Blast Pressure (Analysis Only)

A stiffened compartment with overall dimensions of 24×24×12 in. was modeled for BR-1 computer runs to determine the response of typical aircraft box structure to internal blast pressures. The structure was stiffened by angles in all corners of the structure and by tee sections in one direction only in the top and bottom surfaces of the structure to achieve four bays in both the top and bottom of the structure.

The location of the explosion was selected at the center of the enclosed volume. Because of the symmetrical structure and the location of the explosion, only ⅛ of the structure (Fig. 2) was modeled for the BR-1 computer analysis. In the math model (Fig. 2), there were 79 plate finite elements of which 40 were on the top surface, 24 were on the back surface, and 15 were on the side surface. There were a total of 26 beam elements (to model the stiffeners) in the math model for which adjacent node points along the lines BC, CD, CE, AB, and FG were connected in defining the location of the beam elements. Note that in Fig. 2 there are portions of two bays in the top surface, namely ABGF and FGCE.

In the math model, the skin and stiffener material was characterized by a weight density of 0.0965 lb/in.[3] and a conventional elastic-perfectly plastic engineering stress-strain curve with Young's modulus of 10.4×10^6 psi, Poisson's ratio of 0.3333, a yield strain of 0.00398 in./in., and an ultimate strain of 0.100 in./in. The skin thickness was 0.040 in.

Two problems, which differed only in the description of the projectile threat, were executed with the BR-1 code with the math model of Fig. 2. In one problem, the projectile was characterized by the properties of projectile A in Table 1; in the other problem, projectile C of Table 1 was used. In these two problems, the interaction of fragments with the structural compartment was not considered. Projectile C is a much higher energy explosive than projectile A. From the BLAST code pressure subroutine in the BR-1 code, the BR-1

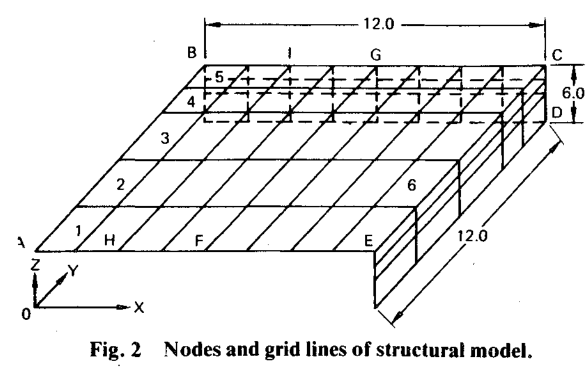

Fig. 2 Nodes and grid lines of structural model.

Table 1 Blast code input data characteristics of two projectiles

Item	Projectile	
	A	C
Charge Weight (lb)	0.0307	.36
Explosive Composition	.74 RDX/.22 Al/.04 Wax	.74 RDX/.22 Al/.04 Wax
Case Length/Diameter Ratio	2.76	3.90
Case/Charge Weight Ratio	5.98	10.64

Standard room environmental conditions were taken as

Chamber pressure (Psia) = 14.70
Chamber temperature (C°) = 20.
Altitude (Kft.) = 0

prediction is that projectile C will produce a 586 psi confined gas overpressure in the enclosed structure whereas projectile A will produce only an 87.6 psi confined gas overpressure in the enclosed structure.

The prediction of the BR-1 code is that projectile C will produce station failure (i.e., structural failure through the skin thickness) much sooner than projectile A. With blast pressure loading by projectile C, station failure occurred in plate elements no. 1, 2, 3, and 4 (see Fig. 2) at 68 μsec after the explosion. However, with blast pressure loading by projectile A, station failure occurred in element No. 6 at 459 μsec after the explosion. From the arrival of the blast pressure at the structure until station failure occurred in the structure, there were a total of eight time increments of numerical integration in the case of the explosion of projectile C and 69 time increments in the case of the explosion of projectile A.

The outward deflection (i.e. the Z deflection) response history of point H, which is midway between stiffeners, after the detonation of threats A and C (i.e., projectiles A and C) is in Fig. 3. The deflection curves (Fig. 3) and stress curves (Fig. 4) are terminated at the time when station failures were predicted in the BR-1 computer execution.

Bending effects play a greater role near the edges of the enclosed structural volume than in portions of the structure not in the vicinity of the edges. Bending effects may be deduced from Fig. 4, in which the s_y stress at the center of element No. 5 and in the upper and lower surface of that element is depicted. The s_y stress is in the length direction of the bay. In the case of projectile A, it is deduced from Fig. 4 that the stress response is initially and finally of the membrane type, but significant bending occurs during a portion of the response. In the case of projectile C, it is deduced from Fig. 4 that the response is essentially exclusive of the membrane type.

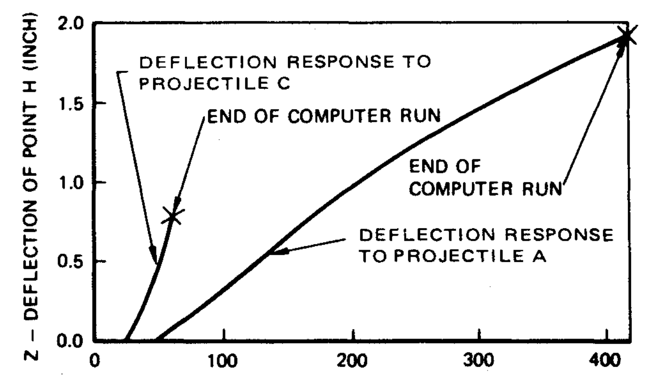

Fig. 3 Deflection of the center of a bay.

Enclosed Compartment Responses to Internal Blast Pressure (Analysis and Test)

A test was conducted in which an empty 7075-T6 aluminum alloy metallic tank with overall dimensions of 21.6×21.6×21.6 in. was attacked by blast pressures and fragments of a projectile that was detonated statically at the center of the tank.[22] The thickness of all the tank walls was 0.063 in. The projectile has been simulated by projectile A in Table 1.

High-speed motion pictures at 6500 frames/sec were taken during the test. Upon examining the motion pictures frame by frame, it appeared that during the transient response the upper tank wall bulged outward with a maximum deflection of about 2.2 in. The experimental transient deflection response at the center of the upper tank wall is included in Fig. 5.

Inasmuch as the test described above was the only known test of a structural compartment that resulted in quantitative data (even though approximate) that could be used to check compartment response predictions made with the BR-1 code, a finite-element model of the structure was prepared for execution with the BR-1 code. Because of assumptions of symmetry in loading and response, only one eighth of the structure was modeled. Twelve square finite elements and nineteen grid points were used in the structural model (Fig. 6).

A two-line segment stress-strain curve was used with Young's modulus of 10.5×10^6 psi, a proportional limit of 68 ksi, an ultimate strength of 78 ksi, and an ultimate strain of 0.08 in./in. A weight density of 0.100 lb/in.^{3} was used in the analysis.

In the BR-1 analysis, it was predicted that the blast pressure first reached the structural walls at 0.2 msec after the detonation. The positive phase durations of the impulsive blast pressure were predicted to be terminated approximately 0.8 msec after the detonation – the fragments penetrated the structure (from the analysis) approximately 0.5 msec after the detonation.

The maximum computed strain in the BR-1 computer run occurred on the inner surface and at the center of element No. 5. The computed stress history at that point is depicted in Fig. 7. The s_y stress history is neither as nearly sinusoidal or periodic as the deflection history in Fig. 5. The explanation for the loss of periodicity in stress centers on the facts that 1) the structural stresses are redistributed spatially from membrane to combined membrane and bending throughout the structure during the transient response, and 2) the stress-strain curve in combination with the high-intensity loading

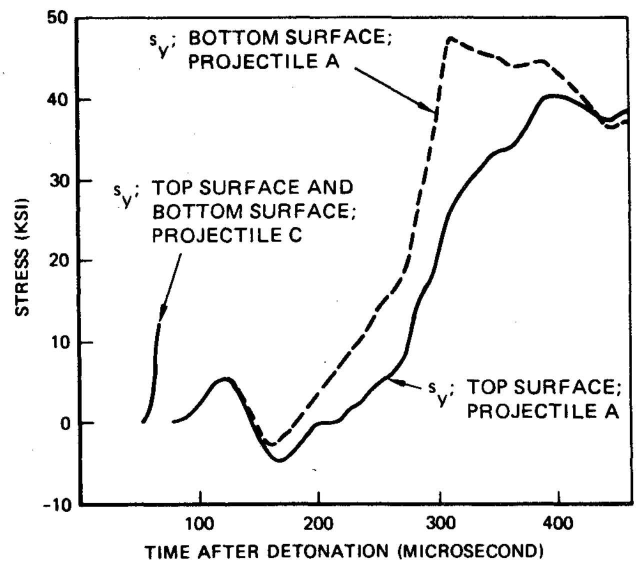

Fig. 4 Stress response on the top and bottom surfaces at the center of element No. 5.

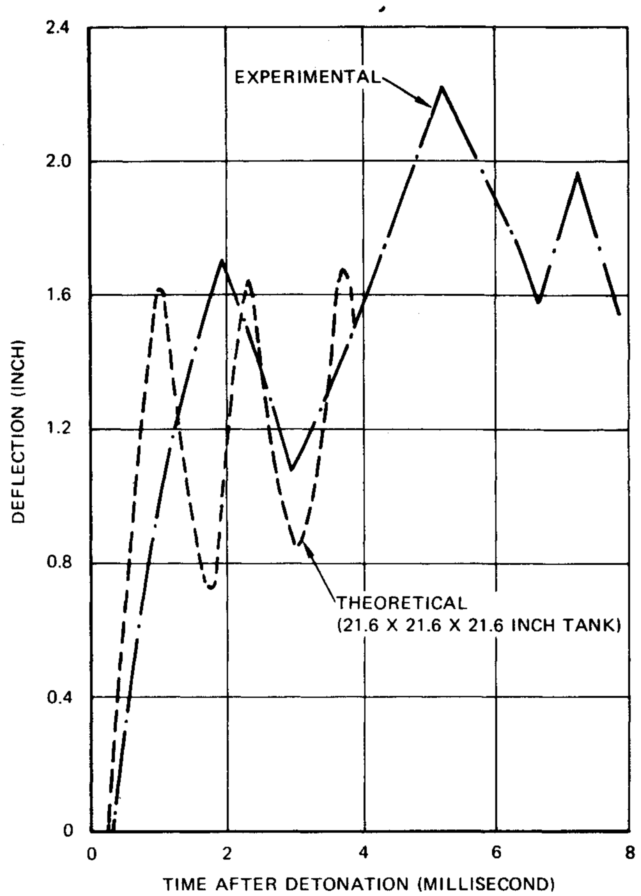

Fig. 5 Outward deflection at the center of the upper wall.

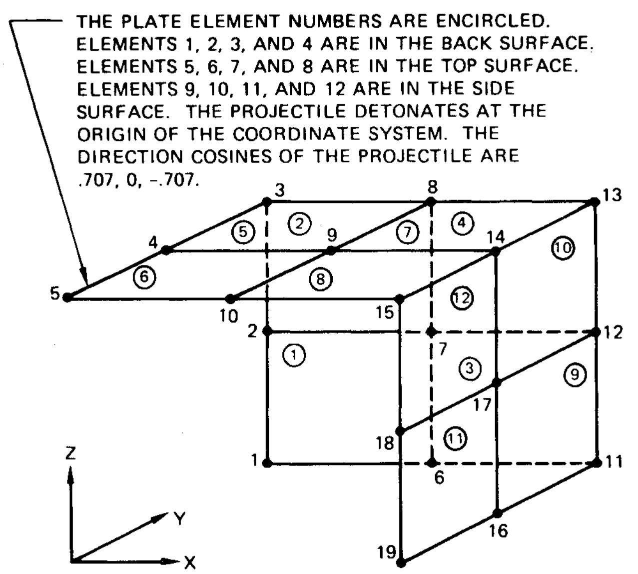

Fig. 6 Structural model for the test tank.

resulted in inelastic strains. Although the strains remained tensile throughout, the corresponding stresses become compressive during a portion of the vibratory response as a result of unloading from an inelastic strain state during portions of the structural response.

Because the deflections and strains appeared to be bounded, the computer run was terminated after 160 time steps in the numerical integration. The computer run required 5 min of cpu time and 3 min of I/0 time on the IBM 370/165 system.

There was no attempt to obtain experimental strains during the test and therefore there are no experimental strain data

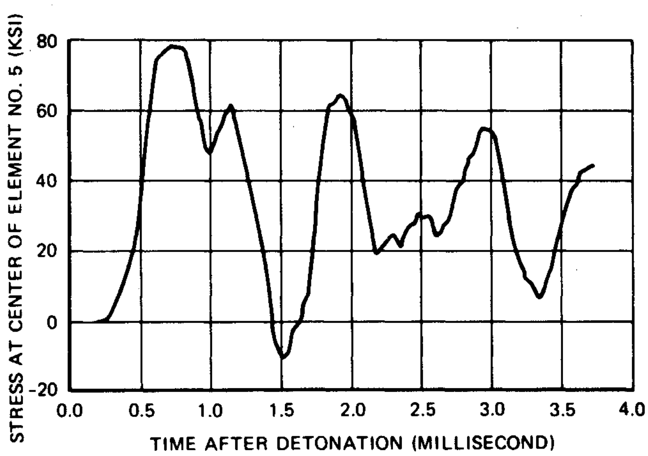

Fig. 7 Transient s_y stress response.

available for comparison with predicted strains from the BR-1 computer run. Inasmuch as no ultimate strain material failures were predicted in the BR-1 analyses, it was predicted on the basis of the BR-1 analyses that the tank walls would contain the explosion insofar as ultimate strain failures (exclusive of the joints) caused by the impulsive loadings were concerned. The test provided positive verification of the accuracy of the prediction of no ultimate strain failures.

The predicted and experimentally obtained outward deflection at the center of the top wall (i.e., at node No. 5 of Fig. 6) is given in Fig. 5. The initial peak excursions in the experimental and the theoretical curves are in good agreement, namely within ±5% of the mean. Deviations between the subsequent experimental and theoretically predicted deflections are attributed partially to the omission in the BR-1 code of the blast pressure loadings from reflections off other surfaces.

Conclusions

The development of the BR-1 computer code has resulted in a new and useful tool for predicting the transient response of metallic skin-rib-stringer type aircraft structures that are subjected to blast pressure and fragmentation loading from the detonation of certain high-explosive projectiles inside the aircraft. The BR-1 code predictions of the transient deflection, strain, and stress response and structural failure may permit the development of a better understanding of the nonlinear structural response problem. The sample problem solutions have demonstrated that the structural response may shift from an almost exclusive membrane response to a combined bending and membrane response and then back to a predominant membrane response because of the large deflections. More test data are needed to verify and/or support predictions obtained with the BR-1 code.

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